USE OF INVERSE PROBLEMS FOR SPATIAL DIAGNOSTICS OF THE HEAT TRANSFER OF AIRCRAFT MODELS

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UDC 536.24.517.946

A theoretical–experimental approach to the diagnostics of nonsteady heat transfer using a scale model of an aircraft prepared by heat-flux sensors is proposed.

Experimental modeling of actual conditions of hypersonic or hyperspeed flow around aircraft of complex spatial form in the presence of nonequilibrium physicochemical reactions in a shock layer is a complex engineering problem. Investigation of aerothermodynamic processes with $M \approx 20$-25 and $Re \approx 10^5$ is often combined with the construction of mathematical models in a shock-compressed, partially dissociated gas. However, experimental confirmation of the heat-flux distribution at the surface is required for the creation of functional and efficient theoretical calculation schemes for the thermal interaction when a real gas flows around three-dimensional bodies, in conditions of strong influence of the physicochemical effects. Moreover, the experimental method must approximate real conditions, which are characterized by considerable temperature, density, and concentration differences of the components of the dissociated gas over the thickness of the shock layer. Therefore, the solution of the general problem of analyzing aircraft thermal conditions is based on complex theoretical–experimental investigation of the spatial heat-transfer pattern at the surface of a body of complex geometric form.

It has been established [1] that, if binary similarity is observed in flow around geometrically similar models, the occurrence of dissociation in the region of nonviscous flow behind a shock wave may be experimentally modeled in the shock layer, and thus the corresponding full-size profiles of the gas-dynamic and thermodynamic flow parameters may be simulated up to the region at the body surface where the influence of dissipative processes becomes significant. Flow around geometrically similar scale models is characterized here by large values of the heat fluxes to the surface, which to some extent imposes constraints on the time of the experiment.

Experimental investigation of the heat transfer in the present work is undertaken on a model with an angle of incidence $\alpha = 30°$ in a hyperspeed aerodynamic apparatus with magnetohydrodynamic acceleration of the air flow [2, 3], allowing velocities of up to 7 km/sec in pulsed conditions with $\tau_{pul} \approx 1$ sec to be reproduced.

A scale model made from VN-2 niobium alloy in the form of a triangular vane with blunt leading edges for the diagnostics of the heat-transfer parameters realized on the stand is prepared on the basis of a sensor apparatus constructed for methods of inverse heat-conduction problems (IHP). The sensitive elements (SE) of the heat-flux sensors (HFS) are also made from niobium alloy, which permits reduction in temperature-field distortion at the surface of the model due to its assembly. In addition, this alloy is distinguished not only by high thermal stability but also by a practically temperature-independent thermal diffusivity $a = \lambda/\rho C$ (m²/sec). This property allows linear formulations of IHP to be considered; these problems may be solved by means of sufficiently simple and workable algorithms.

In the general case, linear IHP are reduced to the solution of a system of algebraic equations [4, 5] for each diagnostic point of the model where the corresponding HFS is placed. In the investigation, the temperature is measured at two widely spaced points over the length of the HFS SE using Chromel–Alumel (C–A) thermocouples. The coefficients of the matrix of the system are determined by the geometric and thermophysical parameters of the mathematical model of heat conduction for the sensor SE in the specified formulation with

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 56, No. 3, pp. 362-368, March, 1989. Original article submitted April 18, 1988.

 0022-0841/89/5603-0244$12.50

known boundary and initial conditions. Since the parameters characterizing the HFS remain constant in the course of the thermal tests, the coefficient matrices for a single discretization of the measurement process $\Delta\tau$ will also be changed. This creates the possibility, in principle, of automatic analysis of the experimental data using a computer in a time scale that is close to real [6].

The algorithms in the present work are based on the solution of the boundary inverse problem for the heat-conduction model consisting of a "wall of constant thickness." The mathematical formulation is as follows:

$$\frac{\partial T(x,\ \tau)}{\partial \tau} = a\,\frac{\partial^2 T(x,\ \tau)}{\partial x^2},\ x \in (0,\ b),\ \tau \in (0,\ \tau_m]. \tag{1}$$

For the specified initial conditions

$$T(x,\ 0) = T_0,\quad x \in [0,\ b] \tag{2}$$

and the experimental temperature information at points x_1 and x_2

$$\begin{aligned} T(x_1,\ \tau) &= f_1(\tau),\quad x_1 \in [0,\ x_2),\quad \tau \in (0,\ \tau_m],\\ T(x_2,\ \tau) &= f_2(\tau),\quad x_2 \in (0,\ b],\quad \tau \in (0,\ \tau_m] \end{aligned} \tag{3}$$

it is required to determine the heat-flux density at the boundary x = 0

$$q(\tau) = -\lambda\,\frac{\partial T(0,\ \tau)}{\partial x},\quad \tau \in (0,\ \tau_m]. \tag{4}$$

Using the Duhamel principle, this problem reduces to the solution of an integral equation of the form [7, 8]

$$\int_0^{\tau_m} q(\tau)\,\frac{\partial \vartheta_1(x_1,\ \tau-\xi)}{\partial \tau}\,d\xi = f_1(\tau) + \int_0^{\tau_m} f_2(\tau)\,\frac{\partial \vartheta_2(x_2-x_1,\ \tau-\xi)}{\partial \tau}\,d\xi, \tag{5}$$

where $\vartheta_1(x,\ \tau)$ is the temperature reaction at point x_1 per unit action at the boundary x = 0; $\vartheta_2(x,\ \tau)$ is the temperature reaction of the body at the point x_1 per unit action at the boundary $x = x_2$.

With the aim of taking account of the dependence of the thermal conductivity on the temperature $\lambda(T)$, within the limits of the linear IHP formulation after Kirchhoff transformation, the model temperature $\Theta(\tau)$ is introduced, with the following definition [4]

$$\Theta_1(\tau) = \frac{1}{\lambda_0}\int_0^{f_1(\tau)} \lambda(T)\,dT,\quad \Theta_2(\tau) = \frac{1}{\lambda_0}\int_0^{f_2(\tau)} \lambda(T)\,dT. \tag{6}$$

As a result of step approximation with a constant integration step $\Delta\tau$, Eq. (5) takes the form

$$\sum_{i=1}^{n} \bar{q}_n \Delta\vartheta_{1,in} = \tilde{\Theta}_n,\quad n = 1,\ 2,\ \ldots,\ m, \tag{7}$$

where

$$\tilde{\Theta}_n = \bar{\Theta}_{1n} + \sum_{i=1}^{n} \bar{\Theta}_{2i}\Delta\vartheta_{2,in};\quad \bar{q}_n = \frac{q_n + q_{n+1}}{2},\quad \Delta\tau = \frac{\tau_m}{m};$$

$$\Delta\vartheta_{j,in} = \Delta\vartheta(x_j,\ \tau_n - \tau_i) - \Delta\vartheta(x_j,\ \tau_n - \tau_{i-1});\ \bar{\Theta}_{j,n} = \frac{\Theta_{j,n} + \Theta_{j,n+1}}{2},\ j = 1,\ 2,$$

or in matrix form

$$A_\Delta q = \Theta. \tag{8}$$

The matrix A_Δ is triangular, with diagonal equality of the elements. The coefficients of the matrix $\Delta\vartheta_{in}$ are determined by the following dependences: for small times $(n - p)\Delta\mathrm{Fo} < 0.4$

$$\Delta\vartheta_{1,in} = \frac{2x_2}{\lambda}\sqrt{\Delta\mathrm{Fo}(n-p)}\sum_{k=0}^{N_1}(-1)^k\left\{i\Phi^*\left[\frac{2k + x_1/x_2}{2\sqrt{\Delta\mathrm{Fo}(n-p)}}\right] - i\Phi^*\left[\frac{2(k+1) - x_1/x_2}{2\sqrt{\Delta\mathrm{Fo}(n-p)}}\right]\right\}\Bigg|_{p=i-1}^{p=i}, \tag{9}$$

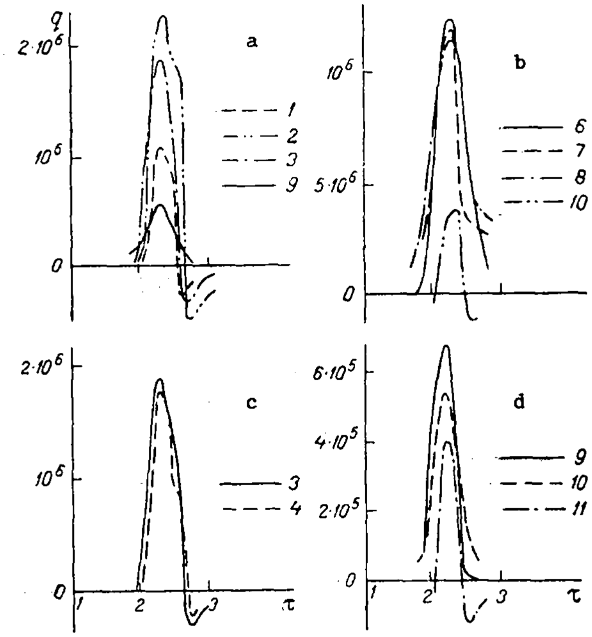

Fig. 1. Derived values of heat-flux density $q(\tau)$ (W/m^2) at the diagnostic points in different cross sections of the model: a) at points 1, 2, 3, 9; b) at points 6, 7, 8, 10; c) at points 3, 4; d) at points 9, 10, 11.

for large $(n - p)\Delta Fo > 0.4$

$$\Delta\vartheta_{1,in} = 8\frac{x_2}{\lambda}\sum_{k=1}^{N_1}\frac{(-1)^k}{\mu_k^2}\left\{\exp\left[-\frac{\mu_k^2\Delta Fo(n-p)}{4}\right]\sin(\mu_k\xi)\right\}\Bigg|_{p=i-1}^{p=i}, \tag{10}$$

where $\mu_k = \pi(2k + 1)$; $\xi = (1/2)(1 - x_1/x_2)$; $\Delta Fo = a\,\Delta\tau/x_2{}^2$.

The coefficients $\Delta\vartheta_{2,in}$ are defined as follows: for small times

$$\Delta\vartheta_{2,in} = \sum_{k=1}^{N_2}(-1)^{k+1}\left\{\Phi^*\left[\frac{2k - x_1/x_2}{2\sqrt{\Delta Fo(n-p)}}\right] + \Phi^*\left[\frac{2(k-1) + x_1/x_2}{2\sqrt{\Delta Fo(n-p)}}\right]\right\}\Bigg|_{p=i-1}^{p=i}, \tag{11}$$

for large times

$$\Delta\vartheta_{2,in} = \frac{4}{\pi}\sum_{k=1}^{N_2}\frac{(-1)^{k+1}}{2k-1}\exp\left[-\frac{\pi^2}{4}(2k-1)^2\Delta Fo(n-p)\right]\times\cos\left[\frac{\pi}{2}(2k-1)(1-x_1/x_2)\right]\Bigg|_{p=i-1}^{p=i}. \tag{12}$$

To obtain a stable solution of Eq. (8), the Tikhonov regularization principle is used [9].

The regularized formulation of the problem reduces to finding the vector q^α giving a minimum of the functional

$$\min_q\left\{\sum_{n=1}^{m}\left(\sum_{i=1}^{n}\bar{q}_i\Delta\vartheta_{1,in} - \tilde{\Theta}_n\right)^2 + \alpha\sum_n(\bar{q}_n - q_{n-1})^2\right\}. \tag{13}$$

The parameter $\alpha > 0$ is determined from the condition of a minimum of the discrepancy, of the form

$$\left|\sum_{n=1}^{m}\left(\sum_{i=1}^{n}\bar{q}_i^\alpha\Delta\vartheta_{1,in} - \tilde{\Theta}_n\right)^2 - \delta_{L_2}^2\right|$$

and agrees with the integral error of the temperature measurements

$$\delta_{L_2}^2 = \int_0^{\tau_m}\sigma^2(\tau)\,d\tau.$$

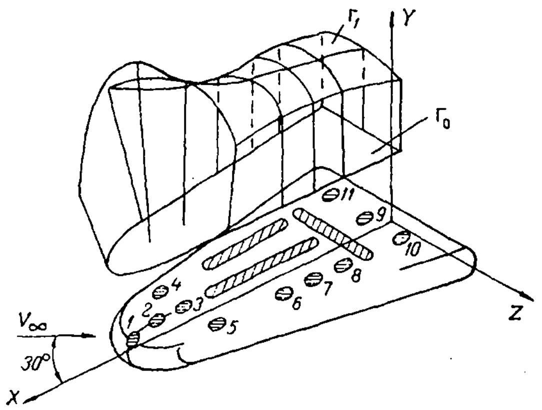

Fig. 2. Scheme of sensor positions and spatial distribution of heat flux at windward surface of the model: Γ_0, initial base surface; Γ_1, surface of flux distribution for time τ = 2.3 sec.

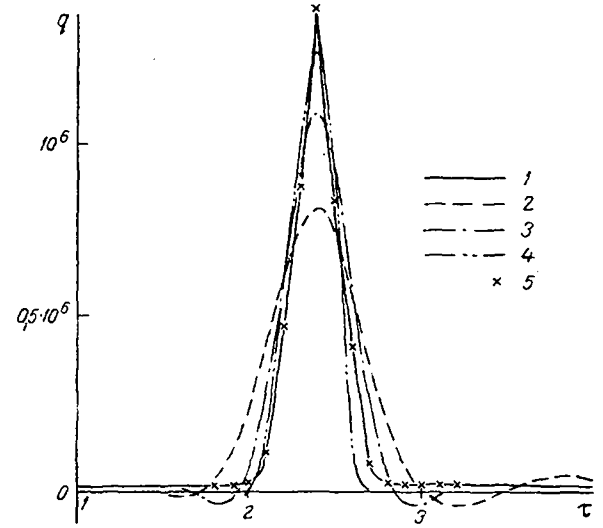

Fig. 3. Comparison of heat fluxes derived by different methods: 1) accurate solution; 2) solution by the iterative regularization method [10], 30 iterations; 3) [10], 100 iterations; 4) [10], 300 iterations; 5) solution by the method proposed here.

After minimization with respect to q, Eq. (13) reduces to a system of linear algebraic equations with a positive-definitive matrix. In operator form, the system of equations takes the form

$$B \cdot q = D, \tag{14}$$

where $B = A_\Delta{}^T \cdot A_\Delta + \alpha C$; $A_\Delta{}^T$ is the transposed matrix; $D = A_\Delta{}^T \cdot \tilde{\Theta} + g$.

The matrix C is a finite-dimensional analog of the smoothing functional taking account, correspondingly, of the smoothness of the first or second derivative of the desired function. The vector g is determined by the boundary conditions [9]. Solving Eq. (14) with the specified condition of smoothness of the desired solution and the chosen value of α, some approximation q^α to the true function $q(\tau)$ is obtained.

The results of deriving the density of local heat fluxes at the diagnostic points in the tested model with analysis of the experimental data using the above algorithm are shown in Fig. 1.

Generalized data on the heat fluxes over the whole of the heated surface of the model allow the pattern of their spatial distribution to be illustrated for the specified time (Fig. 2).

To estimate the effectiveness of use of the given algorithm in analyzing similar expermental investigations characterized by a high intensity with brief heating, the results obtained are compared with solutions by another algorithm. In particular, for a pulsed heat flux specified a priori (τ_{pul} = 0.5 sec, $q_{max} = 1.5 \cdot 10^6$ W/m²) and temperature information calculated at some distance (x_1 = 1.5 mm, x_2 = 4 mm) from the heated surface, the heat flux value derived by the regularization method is compared with the solution obtained by the method of iterative regularization [1]. This analysis shows that the algorithm described here is highly effective. This is confirmed by the small value of the relative error (ε_{max} = 0.2%), in combination with the small amounts of machine time required (~1 min). To obtain a similar accuracy by the iterative method requires more than 400 iterations and around 1-h machine time, even on the high-speed EC-1061 computer. The results of comparing the heat-flux values are shown in Fig. 3.

NOTATION

M, Mach number; Re, Reynolds number; a, λ, thermal diffusivity and thermal conductivity; ρ, density; C, specific heat; T, temperature; x_1, x_2, coordinates; τ, time; τ_m, duration of process; b, thickness; $f_i(\tau)$, experimental temperature values; $q(\tau)$, heat-flux density; Θ, model temperature; α, regularization parameter; δ_{L_2}, integral error of experimental temperature. Indices: max, maximum value; pul, pulse.

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HEAT CONDUCTION IN THE QUASISTEADY HEATING OF MATERIALS

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UDC 536.2+536.2.01

In determining the thermal conductivity of ablative thermally protective materials (APM) by solving the inverse (coefficient) problem of heat conduction (ICP) and calculating the temperature fields in thermally protective coatings based on such materials, investigators encounter several difficulties. The main difficulty relates to the indeterminateness of certain parameters and characteristics in the mathematical model of heat transfer for these materials under conditions of intensive unidirectional heating. Foremost among these parameters and characteristics are the temperature dependences of the density and volumetric specific heat of the given material at high temperatures and the parameters of decomposition of the binder of the APM.

Here, we use the example of the quasisteady regime of heating of APM (steady-state ablation) to analyze the effect of these factors on the thermal conductivity and temperature field of the APM.

With allowance for the heat sinks, we will represent the solution of the direct heat conduction problem (DCP) for quasisteady heating of an APM presented in [1] in the form

$$x = \frac{1}{V}\int\limits_{T}^{T_w} \frac{\lambda(T)\,dT}{\int\limits_{T_0}^{T} C_v(T)\,dT - \Delta H^* \int\limits_{T_0}^{T} \frac{\partial\rho}{\partial T}\,dT - c_g \int\limits_{T_0}^{T}\int\limits_{T_0}^{T} \frac{\partial\rho}{\partial T}\,dT\,dT}. \tag{1}$$

In the heating regime being considered here, the capacity of the internal heat sinks has the form

$$q_v = V\left(\Delta H^* \frac{\partial\rho}{\partial T} - c_g \int\limits_{T_0}^{T} \frac{\partial\rho}{\partial T}\,dT\right)\frac{dT}{dx}.$$

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 56, No. 3, pp. 368-373, March, 1989. Original article submitted April 18, 1988.